Semantic Theory 2014 – Exercise sheet 2

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Exercises are due on Tuesday, May 6, 10:15 a.m.

1. Normal Form Transformations

Every FOL formula can be transformed into an equivalent formula of the form

$Qx_1 \ldots Qx_nA$, where A is quantifier-free ($Qx_1 \ldots Qx_n$ is called “quantifier prefix”, A is called the “matrix” of the formula).

Example: $\forall x \exists y(Fx \wedge Gy)$ is a PNF equivalent of $\forall xFx \wedge \exists xGx$ (Theorems used: 18,23)

1.1 Transform the following formulas into PNF formulas, using theorems from the slides. If necessary, rename bound variables first.

a. $\exists xFx \rightarrow \neg\forall yGy$
b. $\forall x\ \forall y\ (\neg\exists z(Rxyz \vee Fy) \rightarrow \forall xRxy)$
c. $\forall x\ \forall y\ (\forall z(Rxyz \wedge \exists uQxu) \rightarrow \exists vQxv)$

Please, proceed stepwise (in really clear cases you can omit single steps); annotate the steps with the numbers of the theorems used.

Important: I have extended the list of equivalence theorems in the slides. **Please, download and use the revised version of Lecture 1!**

1.2 Using theorems from the slides, eliminate implication and push negation as deep as possible. What you get, is so-called “disjunctive normal form”. Hint: Since only the matrix is modified, you need not copy the quantifier prefix in every step.

2. Type-theoretic representations

Give type-theoretic representations for the following NL sentences.

a. John is taller than Bill
b. John is much taller than Bill
c. Blond is a brighter hair color than brunette.
d. Bill works because the exam is approaching
e. Bill believes that he will pass
f. Bill told Mary that he will take the exam.
g. Bill expects to pass
h. Mary encouraged Bill to take the exam

The examples typically contain one "interesting" expression, which is underlined. Use type inference schemas, as introduced in the lecture, to identify the type of this expression, and to properly construct a type-theoretic formula representing the sentence. Ignore tense, translate "the exam" to a type e expression, assume that the infinitival complements in cases g and h are just standard VPs.

3. Type Constraints

Is it possible to assign types to α, β , and γ in such a way that both (α(β))(γ) and α(β(γ)) are well-formed expressions?

4. Higher-Order Quantification

4.1 Translate Sentence h. to Type Theory:

i. Santa Claus has all attributes of a sadist

4.2 The sentence *Bill is a poor piano player* does not entail that Bill is poor. Therefore attributive adjectives like *poor* cannot be first-order predicates, but must be analyzed as modifiers of first-order predicates. However, the sentence entails that Bill is a piano player: In "x is Adj N" constructions, the adjective typically modifies the semantics of the N in a way that the result is a more restrictive predicate. Specify a type-theoretic formula that expresses this property of restrictiveness for the adjective poor.